X stage flexure design worksheet

Design worksheet for a double parallelogram leaf spring flexure stage with one dof

The feed direction is X, and the slice direction is Y

I'm modeling these beams as fixed-guided.

**Beam parameters**

- E := 70·GPa  
  Elastic modulus  
  E := E
- L_x := 40·mm  
  Beam length  
  L_x := L_x
- t_x := 0.5·mm  
  Beam thickness  
  t_x := t_x
- b := 0.75·in  
  Beam depth  
  b := b
- \( \sigma_{\text{max}} := 200\cdot\text{MPa} \)  
  Max allowable stress  
  \( \sigma_{\text{max}} := \sigma_{\text{max}} \)

**Derived parameters**

The feed direction has a double-parallelogram leaf spring design.

- \( I_{yy} := \frac{1}{12} \cdot b \cdot t_x^3 = 0.198\cdot\text{mm}^4 \)  
  Area moment of inertia along the YY axis (the 'weak' axis)  
  \( I_{yy} := I_{yy} \)
- \( k_{fg} := \frac{12 \cdot E \cdot I_{yy}}{L_x^3} = 2.604\cdot\frac{\text{N}}{\text{mm}} \)  
  Stiffness of a single fixed-guided beam
- \( k_x := 2 \cdot k_{fg} = 5.209\cdot\frac{\text{N}}{\text{mm}} \)  
  Factor of 4, since there are four beams per stage;  
  Factor of 1/2, since there are two stages in series

**Applied loads and resulting displacements, stresses**

- \( F_x := 1\cdot\text{N} \)
- \( \delta_x := \frac{F_x}{k_x} = 0.192\cdot\text{mm} \)  
  Stage displacement

Maximum stress in the beam - assume this occurs at each end of the beam (fixed-guided), at the top and bottom 'fibers'  
(Howell pg 410)

- \( M_y := F_x \cdot \frac{L_x}{2} = 20\cdot\text{N} \cdot \text{mm} \)  
  (at both ends of beam)
- \( \sigma_z := \frac{M_y \cdot t_x}{I_{yy}} = 25.197\cdot\text{MPa} \)

Given a maximum allowable stress, what is my maximum displacement?

- \( x_{\text{max}} := \frac{L_x^2 \cdot \sigma_{\text{max}}}{3 \cdot E \cdot t_x} = 3.048\cdot\text{mm} \)
With two sets of these flexures in series, I can travel twice as far

\[ \delta_{\text{max}} = 2 \cdot x_{\text{max}} = 6.095 \text{ mm} \]