Y axis leadscrew calculations

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This worksheet calculates leadscrew critical buckling loads, critical shaft whip velocity

Material parameters

$E := 200 \cdot GPa$ Young's modulus, Assume steel

$G := \frac{3}{8} \cdot E = 75\,GPa$ Shear modulus

$\rho := 8000 \cdot \frac{kg}{m^3}$ Density

$\sigma_{uts} := 150 \cdot ksi = 1.034 \times 10^3\,MPa$

$\tau_{max} := 0.5 \cdot \sigma_{uts} = 517.107\,MPa$

Geometric parameters

$D := 0.15 \cdot in$ Minor diameter

$L := 4 \cdot in$ Leadscrew length

$a := 2 \cdot in$ Carriage position, measured from one support

$L_o := a$ Leadscrew length (before nut, the part that can buckle)

$b := L - a = 2\,in$ Leadscrew length after nut

$I := \frac{\pi}{64} \cdot D^4 = 10.344\,mm^4$ Bending moment of inertia

$J := \frac{\pi}{32} \cdot D^4 = 20.687\,mm^4$ Polar moment of inertia

$A := \frac{\pi}{4} \cdot D^2 = 11.401\,mm^2$ Cross sectional area

Input loads

$T := 100 \cdot N \cdot mm$

Leadscrew stiffnesses

$$\delta := -\frac{P \cdot a \cdot b \cdot \left(-L^2 + a^2 + b^2\right)}{6 \cdot E \cdot I \cdot L}$$ Assume simply supported beam deflection

$$k_{bend} := \frac{-6 \cdot E \cdot I \cdot L}{a \cdot b \cdot \left(a^2 + b^2 - L^2\right)} = 94.68 \frac{N}{mm}$$

Torsional stiffness

$$k_{\theta y} := \frac{G \cdot J}{L_o} = 30.542 \frac{N \cdot m}{rad}$$ The threaded rod I found have a min UTS of 150 ksi, RC33 hardness, 2A thread (McM 90322A643)

Output velocity

$\omega := 100 \cdot rpm$ Input RPM

$n := 5$ Reduction (if present)

$\omega_{out} := \frac{\omega}{n} = 20\,rpm$ Output RPM

$p := 32 \cdot \frac{1}{in}$ Screw pitch

$v_{out} := \frac{1}{2 \cdot \pi\, p} \cdot \omega_{out} = 0.265 \frac{mm}{s}$ Output velocity

Instabilities

Shaft critical buckling load

$$P_{cr} := \frac{\pi^2 \cdot E \cdot I}{L_o^2} = 7.912\,kN$$

Critical shaft speed

$$\omega_c := (1.875)^2 \cdot \sqrt{\frac{E \cdot I}{A \cdot \rho \cdot L^4}} = 15488.9\,rpm$$

Stresses

Strength of threads (nut and screw)

Torsional stress of leadscrew

$$\tau := \frac{T}{J} \cdot \left(\frac{D}{2} \right) = 9.209\ \text{MPa}$$

Max shear stress is at surface

can also be written as $\tau_{shear} = \frac{16 \cdot T}{\pi \cdot D^3}$

$$T_{max} := \frac{2 \cdot J}{D} \cdot \tau_{max} = 5.615\ \frac{N \cdot m}{rad}$$

Equivalent VM stress

$$\sigma_{VM} = \sqrt{\sigma_{tensile}^{\ 2} + 3 \cdot \tau_{shear}^{\ 2}}$$

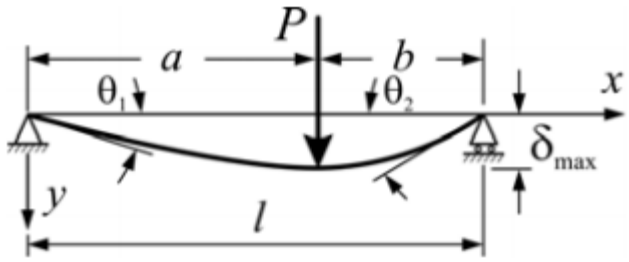
Used if applying tension to screw

Thread engagement to avoid shearing

$$\omega_n = k^2\sqrt{\frac{EI}{A\rho L^4}} \qquad F_{buckle} = \frac{cEI}{L^2}$$

	Cantilevered		Simply Supported		Fixed-Simple		Fixed-Fixed	
mode n	k	c	k	c	k	c	k	c
1	1.875	2.47	3.142	9.87	3.927	20.2	4.730	39.5
2	4.694		6.283		7.069		7.853	
3	7.855		9.425		10.210		10.996	
4	10.996		12.566		13.352		14.137	
n	(2n-1)π/2		nπ		(4n+1)π/4		(2n+1)π/2	

From FUNdaMENTALS, 6-6



$$\delta_{max} = \frac{Pb\left(l^2 - b^2\right)^{3/2}}{9\sqrt{3}\,lEI} \text{ at } x = \sqrt{\left(l^2 - b^2\right)/3}$$

$$\delta = \frac{Pb}{48EI}\left(3l^2 - 4b^2\right) \text{ at the center, if } a > b$$

http://ruina.mae.cornell.edu/Courses/ME4735-2012/Rand4770Vibrations/BeamFormulas.pdf