X stage flexure design worksheet

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Design worksheet for a double parallelogram leaf spring flexure stage with one dof

The feed direction is X, and the slice direction is Y
I'm modeling these beams as fixed-guided.

Beam parameters

$E := 70 \cdot GPa$	Elastic modulus	$E := E$
$L_x := 40 \cdot mm$	Beam length	$L_x := L_x$
$t_x := 0.5 \cdot mm$	Beam thickness	$t_x := t_x$
$b := 0.45 \cdot in$	Beam depth	$b := b$
$\sigma_{xmax} := 200 \cdot MPa$	Max allowable stress	$\sigma_{max} := \sigma_{max}$

Derived parameters

The feed direction has a double-parallelogram leaf spring design.

$I_{yy} := \frac{1}{12} \cdot b \cdot t_x^3 = 0.119 \cdot mm^4$ Area moment of inertia along the YY axis (the 'weak' axis) $I_{yy} := I_{yy}$

$$k_{fg} := \frac{12 \cdot E \cdot I_{yy}}{L_x^3} = 1.563 \cdot \frac{N}{mm}$$

Stiffness of a single fixed-guided beam

$$k_x := 2 \cdot k_{fg} = 3.125 \cdot \frac{N}{mm}$$

Factor of 4, since there are four beams per stage;
Factor of 1/2, since there are two stages in series

Applied loads and resulting displacements, stresses

$F_x := 1 \cdot N$

$$\delta_x := \frac{F_x}{k_x} = 0.32 \cdot mm$$

Stage displacement

Maximum stress in the beam - assume this occurs at each end of the beam (fixed-guided), at the top and bottom 'fibers'
(Howell pg 410)

$$M_y := F_x \cdot \frac{L_x}{2} = 20 \cdot N \cdot mm$$

(at both ends of beam)

$$\sigma_z := \frac{M_y \cdot \frac{t_x}{2}}{I_{yy}} = 41.995 \, MPa$$

Given a maximum allowable stress, what is my maximum displacement?

$$x_{max} := \frac{L_x^2 \cdot \sigma_{xmax}}{3 \cdot E \cdot t_x} = 3.048 \cdot mm$$

With two sets of these flexures in series, I can travel twice as far

$\delta_x := 2 \cdot x_{max} = 6.095 \cdot mm$

Sheet flexure buckling check

Buckling force of an end-loaded plate with shear dominating (from Soemers pg 109)

This is the out-of-plane force that will buckle the blade flexures

$$F_{buckle} := \frac{0.42 \cdot E \cdot b \cdot {t_x}^3}{{L_x}^2} = 26.253\,N$$