Structure Stiffness worksheet

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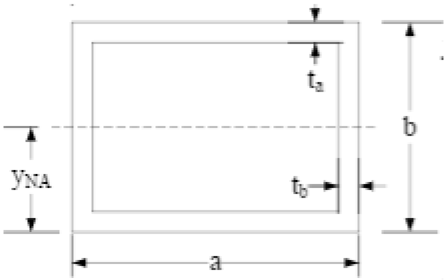
Worksheet to calculate stiffnesses of a box structure. Images from Slocum's FUNdaMENTALS. The results at the bottom feed into the error budget compliance matrices

$b := 1 \cdot in$ $a := 1 \cdot in$

$t_a := 0.125 in$

$t_b := t_a$



Box beam formulae

$$y_{NA} := \frac{b}{2}$$ Neutral axis

$$I_{bending\_weak} := \frac{a \cdot b^3 - (a - 2 \cdot t_b) \cdot (b - 2 \cdot t_a)^3}{12} = 2.371 \times 10^4\ mm^4$$ Bending stiffness about the weak axis

$$I_{bending\_strong} := \frac{b\ a^3 - (a - 2 \cdot t_b)^3 (b - 2 \cdot t_a)}{12} = 2.371 \times 10^4\ mm^4$$ Bending stiffness about the strong axis

$$I_{bending\_square} := \frac{b^4}{12} - \frac{(b - 2 \cdot t_a)^4}{12} = 2.371 \times 10^4\ mm^4$$

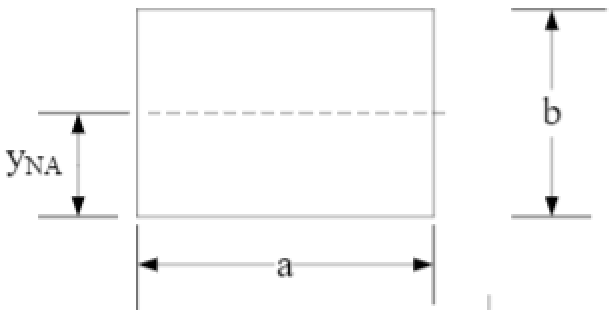
$$I_{torsion} := \frac{2 \cdot t_a \cdot t_b \cdot (a - t_b)^2 \cdot (b - t_a)^2}{a \cdot t_b + b \cdot t_a - t_a^2 - t_b^2} = 3.486 \times 10^4\ mm^4$$

$$I_{torsion\_square} := t_a \cdot (b - t_a)^3 = 3.486 \times 10^4\ mm^4$$ For a square tube with uniform wall thickness

$$\tau_{max\_short} := \frac{\Gamma}{2 \cdot t_b \cdot (a - t_b)(b - t_a)}$$ Max shear on short side

$$\tau_{max\_long} := \frac{\Gamma}{2 \cdot t_a \cdot (a - t_b)(b - t_a)}$$ Max shear on long side

Solid rectangular structure



$y_{NA} := \frac{b}{2}$

$I_{bending\_weak} := \frac{a \cdot b^3}{12} = 3.469 \times 10^{-8}\, m^4$

$I_{bending\_strong} := \frac{b \cdot a^3}{12} = 3.469 \times 10^{-8}\, m^4$

$I_{torsion} := a \cdot b^3 \cdot \left[\frac{16}{3} - 3.36 \cdot \frac{b}{a} \cdot \left(1 - \frac{b^4}{12 \cdot a^4}\right)\right] = 9.379 \times 10^{-7}\, m^4$

$I_{torsion\_square} := 2.25 \cdot b^4$

$\tau_{max} := \frac{4.8 \cdot \Gamma}{a^3}$

Single plate

Plate parameters

$w := 4 \cdot in$ Plate width

$t_1 := 2 \cdot in$ Plate thickness

$$I_{s\_plate\_zz} := \frac{w \cdot t_1^{\,3}}{12} = 110.995\,cm^4$$

Box extrusion frame with two plates

Box parameters

$b := 1 \cdot in$ $a := 1 \cdot in$

$t_a := 0.125 \cdot in$

$t_b := t_a$

Plate parameters

$w := 4 \cdot in$ Plate width

$t_1 := 0.5 \cdot in$ Plate thickness

$A_{plate} := w \cdot t_1 = 12.903\,cm^2$

Geometry

$d_1 := b + t_1 = 1.5\,in$

$d_2 := 3 \cdot in$

$$I_{box\_zz} := \frac{b^4}{12} - \frac{\left(b - 2 \cdot t_a\right)^4}{12} = 2.371 \times 10^4\,mm^4$$

$$I_{plate\_zz} := \frac{w \cdot t_1^{\,3}}{12} = 1.734\,cm^4$$

$$I_{zz\_box\_plate} := 2 \cdot I_{box\_zz} + 2\left[I_{plate\_zz} + A_{plate} \cdot \left(\frac{1}{2} \cdot d_1\right)^2\right] = 101.863\,cm^4$$ Apply parallel axis theorem

$$\frac{I_{zz\_box\_plate}}{I_{s\_plate\_zz}} = 0.918$$ Compare to single thick plate

Plate with rail separators

Rail parameters

$b := 1 \cdot in$ $a := 1 \cdot in$

$t_2 := 0.375 \cdot in$

Plate parameters

Plate parameters unchanged from the box analysis

$w := 4 \cdot in$ Plate width

$t_1 := 0.5 \cdot in$ Plate thickness

$A_{plate} := w \cdot t_1 = 1.29 \times 10^3 \, mm^2$

$$I_{rail\_zz} := \frac{1}{12} \cdot t_2 \cdot b^3 = 1.301 \times 10^4 \, mm^4$$

$$I_{plate\_zz} := \frac{w \cdot t_1^{\,3}}{12} = 1.734 \times 10^4 \, mm^4$$

Apply parallel axis theorem

$$I_{plate\_rail\_zz} := 4 \cdot I_{rail\_zz} + 2 \left[I_{plate\_zz} + A_{plate} \cdot \left(\frac{1}{2} \cdot d_1 \right)^2 \right] = 1.023 \times 10^6 \, mm^4$$

Beam bending loads

$E := 70 \cdot GPa$

$L := 8 \cdot in = 203.2 \, mm$

$d_3 := 2.5 \cdot in = 63.5 \, mm$ load application point

$$k_y := \frac{-6 \cdot E \cdot I_{plate\_rail\_zz} \cdot L}{d_3 \cdot (L - d_3) \cdot \left[d_3^{\,2} + (L - d_3)^2 - L^2 \right]} = 554.854 \, \frac{N}{\mu m}$$

Deflection due to Y force

$$K_{\theta zy} := \frac{3 \cdot E \cdot I_{plate\_rail\_zz} \cdot L}{d_3 \cdot \left(L^2 - 3 \cdot L \cdot d_3 + 2 \cdot d_3^{\,2} \right)} = 6.459 \times 10^7 \, N$$

Slope at load application point due to force

Moment loading stiffness

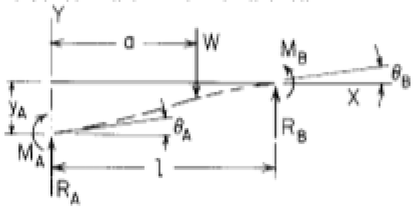
$$K_{\theta z} := \frac{3 \cdot E \cdot I_{plate\_rail\_zz} \cdot L}{L^2 - 3 \cdot L \cdot d_3 + 3 \cdot d_3^{\,2}} = 2.975 \times 10^9 \, \frac{N \cdot mm}{rad}$$

$$k_{yMz} := \frac{3 \cdot E \cdot I_{plate\_rail\_zz} \cdot L}{d_3 \cdot \left(L^2 - 3 \cdot L \cdot d_3 + 2 \cdot d_3^{\,2} \right)} = 6.459 \times 10^7 \, \frac{N \cdot m}{m}$$

Deflection due to Z moment

Beam Bending formulae, from Roark's Formulas for Stress and Strain

1. Concentrated intermediate load



Transverse shear $= V = R_A - W\langle x-a\rangle^0$

Bending moment $= M = M_A + R_A x - W\langle x-a\rangle$

Slope $= \theta = \theta + \frac{M_A x}{EI} + \frac{R_A x^2}{2EI} - \frac{W}{2EI}\langle x-a\rangle^2$

Deflection $= y = y_A + \theta_A x + \frac{M_A x^2}{2EI} + \frac{R_A x^3}{6EI} - \frac{W}{6EI}\langle x-a\rangle^3$

(*Note*: see page 131 for a definition of the term $\langle x-a\rangle^n$.)

1e. Left end simply supported, right end simply supported

$R_A = \frac{W}{l}(l-a) \qquad M_A = 0$

$\theta_A = \frac{-Wa}{6EIl}(2l-a)(l-a) \qquad y_A = 0$

$R_B = \frac{Wa}{l} \qquad M_B = 0$

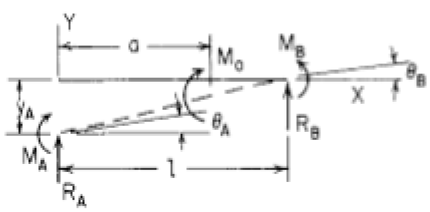
$\theta_B = \frac{Wa}{6EIl}(l^2-a^2) \qquad y_B = 0$

Max $M = R_A a$ at $x = a$; max possible value $= \frac{Wl}{4}$ when $a = \frac{l}{2}$

Max $y = \frac{-Wa}{3EIl}\left(\frac{l^2-a^2}{3}\right)^{3/2}$ at $x = l - \left(\frac{l^2-a^2}{3}\right)^{1/2}$ when $a < \frac{l}{2}$; max possible value $= \frac{-Wl^3}{48EI}$ at $x = \frac{l}{2}$ when $a = \frac{l}{2}$

Max $\theta = \theta_A$ when $a < \frac{l}{2}$; max possible value $= -0.0642\frac{Wl^2}{EI}$ when $a = 0.423l$

3. Concentrated intermediate moment



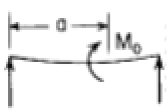
Transverse shear $= V = R_A$

Bending moment $= M = M_A + R_A x + M_o\langle x-a\rangle^0$

Slope $= \theta = \theta_A + \frac{M_A x}{EI} + \frac{R_A x^2}{2EI} + \frac{M_o}{EI}\langle x-a\rangle$

Deflection $= y = y_A + \theta_A x + \frac{M_A x^2}{2EI} + \frac{R_A x^3}{6EI} + \frac{M_o}{2EI}\langle x-a\rangle^2$

3e. Left end simply supported, right end simply supported



$R_A = \frac{-M_o}{l}$

$\theta_A = \frac{-M_o}{6EIl}(2l^2 - 6al + 3a^2)$

$M_A = 0 \qquad y_A = 0$

$R_B = \frac{M_o}{l}$

$\theta_B = \frac{M_o}{6EIl}(l^2 - 3a^2)$

$M_B = 0 \qquad y_B = 0$

Max $+M = \frac{M_o}{l}(l-a)$ just right of $x = a$; max possible value $= M_o$ when $a = 0$

Max $-M = \frac{-M_o a}{l}$ just left of $x = a$; max possible value $= -M_o$ when $a = l$

Max $+y = \frac{M_o(6al - 3a^2 - 2l^2)^{3/2}}{9\sqrt{3}EIl}$ at $x = (2al - a^2 - \frac{2}{3}l^2)^{1/2}$ when $a > 0.423l$; max possible value $= 0.0642\frac{M_o l^2}{EI}$ at $x = 0.577l$ when $a = l$ (*Note*: There is no positive deflection if $a < 0.423l$)