Bushing linear stage stiffness and accuracy worksheet

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Assume a 3-bushing layout (two on one rail, one on the other)
Nomenclature: "Rail 1" has 1 bushing, "rail 2" has 2 bushings. Easy, no?

Carriage, bearing, and rail parameters

Bearing spacing

$d_{22} := 1 \cdot in$ Spacing between bearings on rail 2

$d_{12} := 1 \cdot in$ Spacing between rails

$L_b := 1 \cdot in$ Length of individual bearing

$t_b := 0.0625 in$ Thickness of sleeve bushing

$E_b := 100 \cdot GPa$ Bearing young's modulus

$\mu := 0.2$ Coefficient of friction between bearing and rail

Rail parameters

$E := 200 \cdot GPa$ Steel rods

- Assume rods are not necessarily the same diameter

$D_1 := 0.5 \cdot in$

$D_2 := 0.5 \cdot in$

Length of rails

$L_1 := 3 \cdot in$

$L_2 := L_1$

Moments of inertia

$$I_1 := \frac{\pi}{64} \cdot D_1^{\,4} = 1.277 \times 10^3 \cdot mm^4$$

$$I_2 := \frac{\pi}{64} \cdot D_2^{\,4} = 1.277 \times 10^3 \cdot mm^4$$

$A_b := D_1 \cdot L_b = 322.58 mm^2$ Projected area of bushing

Rail translational, pitch and yaw stiffness

Lateral Stiffness of guide rails

- For rail 1, assume worst-case load at midspan of rail, simply supported
- For rail 2, assume worst case load centered at midspan, simply supported

$$k_1 := \frac{48 \cdot E \cdot I_1}{L_1^{\,3}} = 27.707 \cdot \frac{N}{\mu m}$$

Rail 2 is tricker since we have to superimpose the deflection due to two loads - look up the solution to the deflection of a beam with two point loads

$$k_2 := \frac{2 \cdot 48 \cdot E \cdot I_2}{2 \cdot L_2^{\,3} - 3 \cdot L_2 \cdot d_{22}^{\,2} + d_{22}^{\,3}} = 32.526 \cdot \frac{N}{\mu m}$$

Bushing translational, pitch and yaw stiffnesses

$$K_{lat} := \frac{E \cdot A_b}{t_b} = 4.064 \times 10^4 \frac{N}{\mu m}$$

$$K_{rot} := \frac{1}{12} \cdot K_{lat} \cdot L_b^{\,2} = 2.185 \times 10^6 \frac{N \cdot m}{rad}$$ From PMD 8.5.8

Axial friction force due to imposed moment

$$M := 0.5 \cdot N \cdot m$$

$$F_f := \frac{3 \cdot \mu}{L_b} \cdot M = 11.811\,N$$