Ball slide stage stiffness and accuracy

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Design formulae from H. Soemers, Design Principles for Precision Mechanisms (or Johnson, Contact Mechanics where indicated)

Material definitions

$E_1 := 200 \cdot GPa$

$G_1 := \frac{3}{8} \cdot E_1 = 75 \cdot GPa$

$\nu_1 := 0.3$

$E_2 := 200 \cdot GPa$

$G_2 := \frac{3}{8} \cdot E_2 = 75 \cdot GPa$

$\nu_2 := 0.3$

$\mu := 0.3$

$\sigma_{allow} := 1000 \cdot MPa$

Geometry definitions

Radii

$r_{11} := \frac{1}{8} \cdot in$

$r_{12} := r_{11}$

$r_{21} := 1000000 \, in$

$r_{22} := r_{21}$

$R := \left(\frac{1}{r_{12}} + \frac{1}{r_{22}} \right)^{-1} = 0.125 \cdot in$ Reduced crowning radius

$r := \left(\frac{1}{r_{11}} + \frac{1}{r_{21}} \right)^{-1} = 0.125 \cdot in$ Reduced roll radius; relative curvature

$\omega := \frac{R}{r} = 1$

Crowning-ratio-dependent factors - valid for $1<\omega<25$

$\alpha_\omega := 0.794 \, \omega^{\frac{11}{24}} = 0.794$

$\beta_\omega := 0.794 \cdot \omega^{\frac{-4}{24}} = 0.794$

$\kappa_{\omega} := 0.63 \cdot \omega^{\frac{-9}{44}} = 0.63$

Stage parameters

Number of balls

$n := 6$

Approximate carriage length

$L := n \cdot 2 \cdot r_{11} + n \cdot r_{11} = 2.25\ \text{in}$

$\sqrt[3]{\frac{1}{2}} = 0.794$

Loading

$F := 1 \cdot N$

Calculations

$$E_r := \left(\frac{1 - \nu_1^{\,2}}{2 \cdot E_1} + \frac{1 - \nu_2^{\,2}}{2 \cdot E_2} \right)^{-1} = 219.78 \cdot \text{GPa}$$

Reduced Young's Modulus

$$G_r := \left(\frac{2 - \nu_1}{G_1} + \frac{2 - \nu_2}{G_2} \right)^{-1} = 22.059 \cdot \text{GPa}$$

Reduced shear modulus

$$a := \alpha_{\omega} \cdot \sqrt[3]{\frac{3 \cdot F \cdot r}{E_r}} = 27.89 \cdot \mu\text{m}$$

Half of the contact width, long axis

$$b := \beta_{\omega} \cdot \sqrt[3]{\frac{3 \cdot F \cdot r}{E_r}} = 27.89 \cdot \mu\text{m}$$

Half of the contact width, short axis

$$q_{max} := \frac{3 \cdot F}{2 \cdot \pi \cdot a \cdot b} = 613.832 \cdot \text{MPa}$$

Maximum contact stress (Johnson 4.32)

$$\delta := \kappa_{\omega} \cdot \sqrt[3]{\frac{9 \cdot F^2}{E_r^{\,2} \cdot r}} = 0.245 \cdot \mu\text{m}$$

Approach of body center to original undeformed surface

$$k := \frac{1}{\kappa_{\omega}} \cdot \sqrt[3]{\frac{8}{3} \cdot F \cdot E_r^{\,2} \cdot r} = 11.782 \cdot \frac{N}{\mu\text{m}}$$

Load-dependent normal stiffness

$$\frac{F}{\delta} = 4.085 \cdot \frac{N}{\mu\text{m}}$$

$$\delta_t := \frac{3\cdot\mu\cdot F\cdot G_r}{16\cdot r}\cdot\left[1-\left(1-\frac{F_t}{\mu\cdot F_n}\right)^{\frac{2}{3}}\right] = \blacksquare$$

Tangential deflection due to tangential force

$$k_t := 8\cdot G_r\cdot r\cdot\left(1-\frac{F_t}{\mu\cdot F_n}\right)^{\frac{1}{3}} = \blacksquare$$

Tangential stiffness

$$\tau_{1max} := 0.47\cdot\frac{F}{\pi\cdot a^2} = 192.334\cdot MPa$$

Principal shear stress (Johnson)

- Assume that material will fail at 0.5*UTS

$$z := 0.57\cdot a = 15.897\cdot\mu m$$

Depth of maximum shear stress

$$q_{allow} := \frac{3}{2}\cdot\sigma_{allow} = 1500\cdot MPa$$

Allowable contact pressure (valid for ductile metals)

Stage stiffnesses

$$k_x := n\frac{4}{\sqrt{2}}\cdot k = 199.949\cdot\frac{N}{\mu m}$$

Rotational stiffness

$$k_{rot} := \frac{1}{12}\cdot k_x\cdot L^2 = 5.442\times10^4\frac{N\cdot m}{rad}$$