Geared stepper motor and leadscrew resolution worksheet

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How fine of a resolution can I get with a stepper motor based system?
How do I deal with stiction?

Gearbox reduction

$n := 14$

Leadscrew parameters

$R := \frac{1}{4} \cdot in$ $\quad TPI := 80 \cdot \frac{1}{in}$

$\alpha := 30 \cdot deg$ Standard V thread (14.5 deg if acme)

$\mu := 0.15$ Coefficient of friction

$l := \frac{1}{TPI} = 0.317 \cdot mm$ Lead angle

$\theta := atan\left(\frac{l}{2 \cdot \pi \cdot R}\right) = 0.456\, deg$

Step resolution

Assume half stepping

$1 < \frac{2 \cdot \pi \cdot R \cdot \mu}{\cos(\alpha)} = 1$ If true, it's self-locking

$\theta_{step} := 0.9 \cdot deg$

$\theta_{gear} := \frac{\theta_{step}}{n} = 0.064\, deg$

Each turn of the screw moves the nut forward by the lead

$n_2 := \frac{l}{360 \cdot deg} = 8.819 \times 10^{-4} \cdot \frac{mm}{deg}$

$\theta_{out} := \theta_{gear} \cdot n_2 = 0.057 \cdot \mu m$

It's on the money in practice, but how well would it work?

Moment required to turn screw, with applied axial force

$F_Z := 10 \cdot N$

$C_{\theta R} := \dfrac{l \cdot \cos(\alpha) + 2 \cdot \pi \cdot R \cdot \mu}{2 \cdot \pi \cdot R \cdot \cos(\alpha) - \mu \cdot l} = 0.181$ PMD (10.8.9)

$R = 6.35 \cdot mm$

Raising a load:

$M_Z := F_Z \cdot C_{\theta R} \cdot R = 11.52 \cdot N \cdot mm$ PMD (10.8.14)

Lowering a load:

$M_Z := F_Z \cdot R \cdot \left(\dfrac{2 \cdot \pi \cdot R \cdot \mu - l \cdot \cos(\alpha)}{2 \cdot \pi \cdot R \cdot \cos(\alpha) + \mu \cdot l} \right) = 10.479 \cdot N \cdot mm$

Efficiency

$\beta := \dfrac{2 \cdot R}{l} = 40$ (PMD 10.8.18)

$\eta := \dfrac{\cos(\alpha) \cdot (\pi \cdot \beta \cdot \cos(\alpha) - \mu)}{\pi \cdot \beta \cdot \cos(\alpha) \cdot (\cos(\alpha) + \pi \cdot \beta \cdot \mu)} = 0.044$

Differential leadscrew displacements

Desired output motion

$x := 6 \cdot mm$

$TPI_1 := 14 \cdot \dfrac{1}{in}$ Coarse pitch

$TPI_2 := 16 \cdot \dfrac{1}{in}$ Fine pitch

$l_1 := \dfrac{1}{TPI_1} = 1.814 \cdot mm$

$l_2 := \dfrac{1}{TPI_2} = 1.587 \cdot mm$ $70 \cdot GPa = 7 \times 10^4 \cdot \dfrac{N}{mm^2}$

$l_e := l_1 - l_2 = 0.227 \cdot mm$

$TPI_e := \dfrac{1}{l_e} = 112 \cdot \dfrac{1}{in}$ Effective pitch

How far must each component travel to get the full range of output motion?

$N_e := \dfrac{x}{l_e} = 26.457$ Number of turns to get full range

$X_1 := N_e \cdot l_1 = 48 \cdot mm$ Distance traveled by coarse pitch screw

$X_2 := -N_e \cdot l_2 = -42 \cdot mm$ Distance nut travels on fine pitch screw