X stage flexure design worksheet

Aaron Ramirez 4/13/2016

Design worksheet for a double parallelogram leaf spring flexure stage with one dof

The feed direction is X, and the slice direction is Y
I'm modeling these beams as fixed-guided.

Beam parameters

$E := 70 \cdot GPa$	Elastic modulus	$E := E$
$L_x := 40 \cdot mm$	Beam length	$L_x := L_x$
$t_x := 0.5 \cdot mm$	Beam thickness	$t_x := t_x$
$b := 0.75 \cdot in$	Beam depth	$b := b$
$\sigma_{xmax} := 200 \cdot MPa$	Max allowable stress	$\sigma_{max} := \sigma_{max}$

Derived parameters

The feed direction has a double-parallelogram leaf spring design.

$I_{yy} := \frac{1}{12} \cdot b \cdot t_x^{\,3} = 0.198\,mm^4$ Area moment of inertia along the YY axis (the 'weak' axis) $I_{yy} := I_{yy}$

$k_{fg} := \frac{12 \cdot E \cdot I_{yy}}{L_x^{\,3}} = 2.604 \frac{N}{mm}$ Stiffness of a single fixed-guided beam

$k_x := 2 \cdot k_{fg} = 5.209 \frac{N}{mm}$ Factor of 4, since there are four beams per stage;
Factor of 1/2, since there are two stages in series

Applied loads and resulting displacements, stresses

$F_x := 1 \cdot N$

$\delta_x := \frac{F_x}{k_x} = 0.192\,mm$ Stage displacement

Maximum stress in the beam - assume this occurs at each end of the beam (fixed-guided), at the top and bottom 'fibers'
(Howell pg 410)

$M_y := F_x \cdot \frac{L_x}{2} = 20\,N \cdot mm$ (at both ends of beam)

$\sigma_z := \frac{M_y \cdot \frac{t_x}{2}}{I_{yy}} = 25.197\,MPa$

Given a maximum allowable stress, what is my maximum displacement?

$x_{max} := \frac{L_x^{\,2} \cdot \sigma_{xmax}}{3 \cdot E \cdot t_x} = 3.048\,mm$

With two sets of these flexures in series, I can travel twice as far

$\delta_x := 2 \cdot x_{max} = 6.095 \cdot mm$