X decoupler design worksheet

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Design worksheet for a 4-dof leadscrew nut flexure stiff in the axial and torsional windup directions

This is the analytical modeling for a two-stage flexure which decouples misalignments from the output stage. I refer to the intermediate stage as stage 1 and the output as stage 2, blades are subscripted with b and wires with w.

Beam parameters

$E := 70 \cdot GPa$	Elastic modulus	$E := E$
$L_x := 40 \cdot mm$	Beam length	$L_x := L_x$
$t_x := 0.5 \cdot mm$	Beam thickness	$t_x := t_x$
$b := 0.45 \cdot in$	Beam depth	$b := b$
$\sigma_{xmax} := 200 \cdot MPa$	Max allowable stress	$\sigma_{max} := \sigma_{max}$

- Stage 1 definitions

  Blade

  $l_{1b} := 10 \cdot mm$

  $b_{1b} := 15 \cdot mm$

  $t_{1b} := 0.5 \cdot mm$

  Wire

  $l_{1w} := l_{1b} = 10 \cdot mm$

  $b_{1w} := 0.5 \cdot mm$

  $t_{1w} := 0.5 \cdot mm$

  Spacing between blade plane and wire plane

  $n_{1w} := 2$ Number of wires

  $d_{1bw} := 15 \cdot mm$

  $d_{1ww} := 10 \cdot mm$

- Stage 2 definitions

  Blade

  $l_{2b} := 10 \cdot mm$

  $b_{2b} := b_{1b} = 15 \cdot mm$

  $t_{2b} := 0.5 \cdot mm$

  Wire

  $l_{2w} := 8 \cdot mm$

  $b_{2w} := b_{1w} = 0.5 \cdot mm$

  $t_{2w} := b_{2w} = 0.5 \cdot mm$

  Flexure spacings

  $d_{2ww} := 10 \cdot mm$ Distance between wires (height of stage)

$d_{2bb} := 10 \cdot mm$ Distance between blades (width of stage)

Derived parameters

- Cross sectional areas

$A_{1b} := t_{1b} \cdot b_{1b} = 7.5 \cdot mm^2$

$A_{1w} := t_{1w} \cdot b_{1w} = 0.25 \cdot mm^2$

$A_{2b} := t_{2b} \cdot b_{2b} = 7.5 \cdot mm^2$

$A_{2w} := t_{2w} \cdot b_{2w} = 0.25 \cdot mm^2$

- Moments of inertia

$I_{1byy} := \frac{1}{12} \cdot t_{1b} \cdot b_{1b}^3 = 140.625 \cdot mm^4$

$I_{1wyy} := \frac{1}{12} \cdot t_{1w} \cdot b_{1w}^3 = 5.208 \times 10^{-3} \cdot mm^4$

$I_{1bxx} := \frac{1}{12} \cdot t_{1b}^3 \cdot b_{1b} = 0.156 \cdot mm^4$

$I_{1wxx} := \frac{1}{12} \cdot t_{1w}^3 \cdot b_{1w} = 5.208 \times 10^{-3} \cdot mm^4$

$I_{2byy} := \frac{1}{12} \cdot t_{2b} \cdot b_{2b}^3 = 140.625 \cdot mm^4$

$I_{2wyy} := \frac{1}{12} \cdot t_{2w} \cdot b_{2w}^3 = 5.208 \times 10^{-3} \cdot mm^4$

$I_{2bxx} := \frac{1}{12} \cdot t_{2b}^3 \cdot b_{2b} = 0.156 \cdot mm^4$

$I_{2wxx} := \frac{1}{12} \cdot t_{2w} \cdot b_{2w}^3 = 5.208 \times 10^{-3} \cdot mm^4$

$I_{yy} := \frac{1}{12} \cdot b \cdot t_x^3 = 0.119 \cdot mm^4$ Area moment of inertia along the YY axis (the 'weak' axis)

$k_{fg} := \frac{12 \cdot E \cdot I_{yy}}{L_x^3} = 1.563 \cdot \frac{N}{mm}$ Stiffness of a single fixed-guided beam

$k_x := 2 \cdot k_{fg} = 3.125 \cdot \frac{N}{mm}$ Factor of 4, since there are four beams per stage;
Factor of 1/2, since there are two stages in series

Applied loads and resulting displacements, stresses

$F_x := 1 \cdot N$

$\delta_x := \frac{F_x}{k_x} = 0.32 \cdot mm$ Stage displacement

Maximum stress in the beam - assume this occurs at each end of the beam (fixed-guided), at the top and bottom 'fibers'
(Howell pg 410)

$$M_y := F_x \cdot \frac{L_x}{2} = 20 \cdot N \cdot mm$$ (at both ends of beam)

$$\sigma_z := \frac{M_y \cdot \frac{t_x}{2}}{I_{yy}} = 41.995 \cdot MPa$$

Given a maximum allowable stress, what is my maximum displacement?

$$x_{max} := \frac{L_x^{\,2} \cdot \sigma_{xmax}}{3 \cdot E \cdot t_x} = 3.048 \cdot mm$$

With two sets of these flexures in series, I can travel twice as far

$$\delta_x := 2 \cdot x_{max} = 6.095 \cdot mm$$

Stiffness between ground and intermediate stage

X Blade bends about its stiff axis

$$k_{1bx} := \frac{12 \cdot E \cdot I_{1byy}}{l_{1b}^{\,3}} = 118.125 \cdot \frac{N}{\mu m}$$

Dont forget shear correction factor!

$$k_{1wx} := \frac{12 \cdot E \cdot I_{1wyy}}{l_{1w}^{\,3}} = 4.375 \times 10^{-3} \cdot \frac{N}{\mu m}$$

$$k_{1x} := 2 \cdot k_{1wx} + k_{1bx} = 118.134 \cdot \frac{N}{\mu m}$$

Y Stiffness in the blade and wire axial direction

$$k_{1by} := \frac{E \cdot A_{1b}}{l_{1b}} = 52.5 \cdot \frac{N}{\mu m}$$

$$k_{1wy} := \frac{E \cdot A_{1w}}{l_{1w}} = 1.75 \cdot \frac{N}{\mu m}$$

$$k_{1y} := k_{1by} + n_{1w} \cdot k_{1wy} = 56 \cdot \frac{N}{\mu m}$$

Z Blade bends about its weak axis

$$k_{1bz} := \frac{12 \cdot E \cdot I_{1bxx}}{l_{1b}^{\,3}} = 0.131 \cdot \frac{N}{\mu m}$$

$$k_{1wz} := \frac{12 \cdot E \cdot I_{1wxx}}{l_{1w}^{\,3}} = 4.375 \times 10^{-3} \cdot \frac{N}{\mu m}$$

$$k_{1z} := 2 \cdot k_{1wz} + k_{1bz} = 0.14 \cdot \frac{N}{\mu m}$$

θx Stage tries to twist about leadscrew axis (X)

$$k_{1\theta x} := \frac{k_{1by} \cdot n_{1w} \cdot k_{1wy}}{k_{1by} + n_{1w} \cdot k_{1wy}} \cdot d_{1bw}^{\;2} = 738.281 \cdot \frac{N \cdot m}{rad}$$

θy Stage yaws about Y

$$k_{1\theta y} := \frac{k_{1bx} \cdot n_{1w} \cdot k_{1wx}}{k_{1bx} + n_{1w} \cdot k_{1wx}} \cdot d_{1bw}^{\;2} = 1.969 \cdot \frac{N \cdot m}{rad}$$

Blade tries to bend against its stiff direction

θz Stage pitches about Z

$$k_{1b\theta z} := \frac{1}{12} \cdot t_{1b} \cdot b_{1b}^{\;3} \cdot \frac{E}{l_{1b}} = 984.375 \cdot \frac{N \cdot m}{rad}$$

Blade contribution:

Moment bending about cantilever tip

$$k_{1w\theta z} := \frac{k_{1wy} \cdot k_{1wy}}{k_{1wy} + k_{1wy}} \cdot d_{1ww}^{\;2} = 87.5 \cdot \frac{N \cdot m}{rad}$$

Wire contribution: push-pull pair

$$k_{1\theta z} := k_{1b\theta z} + k_{1w\theta z} = 1.072 \times 10^{3} \cdot \frac{N \cdot m}{rad}$$

Combine the equivalent stiffnesses

Stiffnesses between intermediate stage and output

X Sum of blades bending in stiff direction and wires bending

$$k_{2bx} := \frac{12 \cdot E \cdot I_{2byy}}{l_{2b}^{\;3}} = 118.125 \cdot \frac{N}{\mu m}$$

Individual blade bending in stiff direction

Apply shear correction factor!

$$k_{2wx} := \frac{12 \cdot E \cdot I_{2wyy}}{l_{2w}^{\;3}} = 8.545 \times 10^{-3} \cdot \frac{N}{\mu m}$$

$$k_{2x} := k_{2bx} + k_{2wx} = 118.134 \cdot \frac{N}{\mu m}$$

Y Sum of blade bending in compliant direction and wire bending

$$k_{2by} := \frac{12 \cdot E \cdot I_{2bxx}}{l_{2b}^{\;3}} = 131.25 \cdot \frac{N}{mm}$$

$$k_{2wy} := \frac{12 \cdot E \cdot I_{2wxx}}{l_{2w}^{\;3}} = 8.545 \cdot \frac{N}{mm}$$

$$k_{2y} := 2k_{2by} + 8 \cdot k_{2wy} = 0.331 \cdot \frac{N}{\mu m}$$

Z Stiffness in the axial direction

$$k_{2bz} := \frac{E \cdot A_{2b}}{l_{2b}} = 52.5 \cdot \frac{N}{\mu m}$$

$$k_{2wz} := \frac{E \cdot A_{2w}}{l_{2w}} = 2.188 \cdot \frac{N}{\mu m}$$

$$k_{2z} := 2 \cdot k_{2bz} + 8 \cdot k_{2wz} = 122.5 \cdot \frac{N}{\mu m}$$

θ_x Stiffness in the leadscrew windup direction

Resisted by axial stiffness of wires and bending stiffness of blade

Contribution of wire flexures

$$k_{2w\theta x} := k_{2wz} \cdot \left(\frac{1}{2} \cdot d_{2ww}\right)^2 = 54.688 \cdot \frac{N \cdot m}{rad}$$

Contribution of blade flexures

$$k_{2b\theta x} := k_{2by} \cdot \left(\frac{1}{2} \cdot d_{2bb}\right)^2 = 3.281 \cdot \frac{N \cdot m}{rad}$$

$$k_{2\theta x} := 8k_{2w\theta x} + 2 \cdot k_{2b\theta x} = 444.063 \cdot \frac{N \cdot m}{rad}$$

θ_y Stage 2 roll direction stiffness

Wire contribution: bending

$$k_{2w\theta y} := k_{2wx} \cdot \left(\frac{1}{2} \cdot d_{2bb}\right)^2 = 0.214 \cdot \frac{N \cdot m}{rad}$$

Blade contribution

$$k_{2b\theta y} := \frac{1}{12} \cdot t_{2b} \cdot b_{2b}{}^3 \cdot \frac{E}{l_{2b}} = 984.375 \cdot \frac{N \cdot m}{rad}$$

$$k_{2\theta y} := 8 \cdot k_{2w\theta y} + 2 \cdot k_{1b\theta z} = 1.97 \times 10^3 \cdot \frac{N \cdot m}{rad}$$

θ_z Stage 2 pitching stiffness

$$k_{2w\theta z} := k_{2wy} \cdot \left(\frac{1}{2} \cdot b_{2b}\right)^2 = 0.481 \cdot \frac{N \cdot m}{rad}$$

Wire flexures bend

$$k_{2b\theta z} :=$$

$k_{2\theta z} := 8 \cdot k_{2w\theta z} + \blacksquare$

Resulting output stiffnesses

Constrained directions

$k_x := \blacksquare$

$K_{\theta x} := \blacksquare$

Unconstrained directions

$k_z := \blacksquare$

$k_y := \blacksquare$

$K_{\theta y}$

$K_{\theta z}$